Effect of Heat Transfer on the Peristaltic MHD Flow of a Jeffrey Fluid in An Inclined Channel

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***Abstract*— Peristaltic transport of a conducting Jeffrey fluid in an inclined channel with heat transfer is studied under long wave length and low Reynolds number approximations. An exact solution is presented for the temperature field. The expressions for axial velocity, axial pressure gradient, Stream line and coefficient of heat transfer have been obtained analytically. The effect of various parameters on the flow characteristics are discussed with the help of graph.**

***Key words* — Jeffrey fluid, MHD flow, heat transfer, inclined channel.**

I. INTRODUCTION

Peristaltic pumping is a form of fluid transport which occurs in a biological system. The study of peristaltic motion has gained considerable interest because of its extensive role in transporting many physiological fluid in the body in various situations such as urine transport from the kidney to bladder, transport of the spermatozoa in the ducts afferents of male reproductive tract, the moment of chyme in the gastrointestinal tract, swallowing of food through esophagus and vasomotion of small blood vessels. Many mechanical devices have been designed on the peristaltic pumping to transport fluids without internal moving parts, for example, the transport of slurries, sensitive or corrosive fluids, sanitary fluid and noxious fluids in the nuclear industries. The literature on this topic is quite extensive. Mention may be made to some recent theoretical and experimental contributions Shapiro and Jaffrin [1], Hayat and Sajid [2], El Misery et al.[3],Yin and Fung[4,5], Pzrikidis [6], Shukla and gupta[7], Vjravelu et al.[8,9], Raju and Devanathan[10] in the field for Newtonian fluids. Such approximation is true in the ureter but it fails to give an adequate understanding of food mixing and chyme moment in the intestine, fuel slurries, flow of plasma, flow of mercury amalgams, and lubrication with heavy oils and greases etc. Also, the assumptions that most of the physiological fluid behave like Newtonian fluid is not true in reality. With all these facts in mind, it is clear that non Newtonian fluid plays an indispensable role in peristaltic flow problem.

A number of researchers have been discussed the effect of magnetic field on peristaltic flow Mekheimer [11], Harikrishana and Subbareddyy [12], Suryanarayana Reddy et al. [13], Subba Reddy and Gangadhar [14], Elshahed and Haroun [15], Kothadapani [16, 17], Affi and Gad[118], Krishana kumari and Raman Murthy [19], Nadeem and Akbar [21], due to its applications in bio engineering and medical devices. Specially, magnetic wound or cancer treatment causes hyperthermia, bleeding reduction during surgeries, and targeted transport of drugs using magnetic particles as drug carries are few examples.

The interaction of peristalsis in connection with heat transfer has also received some attention Nadeem and Akbar [21], Srinivas and Gayatri [22], Hayat and Heena [23], Hayat and Ali [24], Hayat et al.[25] as it might be relevant in processes like hemodialysis and oxygenation. Rathod and Mahadev [26] have studied the effect of thickness of porous material on the peristaltic pumping of Jeffrey fluid with non-erodible porous ling wall. Effects of magnetic field and an endoscope on peristaltic motion is studied by Rathod and Asha [27]. Rathod and Pallavi [28] have discussed the influence of wall properties on MHD peristaltic transport of dusty fluid.

The present article studies the effect of heat transfer on peristaltic MHD flow of Jeffrey fluid in an inclined channel. The governing equations of Jeffrey fluid in Cartesian co-ordinate have been modeled. The equations are simplified using long wave length and low Reynolds number approximations. The closed form of solution for velocity field and temperature are obtained. The influence of various parameters on the flow characteristics, the temperature and heat transfer coefficient are discussed through graph.

2. FORMULATION OF THE PROBLEM

Consider the peristaltic pumping of a Jeffrey fluid with heat transfer in an inclined channel of half-width ‘a’. A longitudinal train of progressive sinusoidal wave takes place on the upper and lower wall of the channel. We further assume that the fluid is electrically conducting. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The Reynolds number is taken small so that the induced magnetic field is neglected. For simplicity, we restrict our discussion to the half width of the channel as shown in figure .(1)

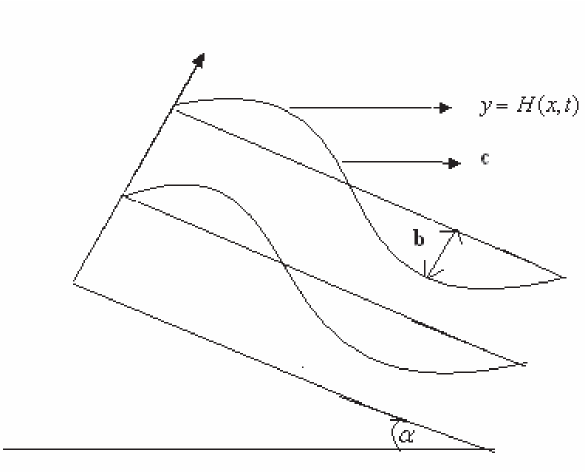

Figure.1. Physical model

The wall deformation is given by

$$H(x,t) = a + bCos\frac{2\pi}{\lambda}(X - ct) \qquad (1)$$

where *b* is amplitude of the waves and *λ* is the wave length.
The constitutive equations for an incompressible Jeffrey fluid are

$$\bar{T} = -P\bar{I} + \bar{S} \qquad (2)$$

$$\bar{S} = \frac{\mu}{1+\lambda_1}(\dot{\gamma} + \lambda_2\ddot{\gamma}) \qquad (3)$$

where $\bar{T}$ and $\bar{S}$ are Cauchy stress tensor and extra stress tensor, $\bar{p}$ is the pressure, $\bar{I}$ is the identity tensor, μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation.

Under the assumptions that the channel length is an integral multiple of the wave length λ and the pressure difference across the ends of the channel is a constant, the flow is inherently unsteady in the laboratory frame $(\bar{X}, \bar{Y})$ and become steady in the wave frame $(\bar{x}, \bar{y})$ which is moving with velocity '*c*' along the wave. The transformation between these two frames is given by

$$x = \bar{X} - ct, y = \bar{Y}, u = \bar{U}, v = \bar{V} \qquad (4)$$

where *U* and *V* are velocity components in the laboratory frame and *u* and *v* are the velocity components in the wave frame. In the many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the wavelength is infinite, the flow is of Poiseuille type at each local cross-section.
Introducing the non-dimensional quantities

$$x = \frac{2\pi\bar{x}}{\lambda}, y = \frac{\bar{y}}{a}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c\delta}, \delta = \frac{2\pi a}{\lambda}, p = \frac{2\pi a^2\bar{p}}{\mu c\lambda},$$

$$t = \frac{2\pi a^2}{\lambda}, S = \frac{a}{\mu c}\bar{S}, \phi = \frac{b}{a}, G = \frac{\rho g a^2}{\mu c}, \nu = \frac{\mu}{\rho}, R = \frac{ca}{\nu}, Pr = \frac{\mu c}{k},$$

$$Ec = \frac{c^2}{c_p(T_1 - T_0)}, T = \theta(T_1 - T_0) + T_0, u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$$

where R is the Reynolds number, *G* is the gravitational parameter, Ec is the Eckert number and Pr is the Prandtl number.
The equations governing the flow become

$$\frac{\partial u}{\partial x} + \delta\frac{\partial u}{\partial y} = 0, \qquad (5)$$

$$\delta R\left[(u+1)\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\sigma^2(u+1)+\rho g\sin\alpha \qquad (6)$$

$$\delta^3 R\left[(u+1)\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial x}+\delta^2\frac{\partial S_{xy}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y}-\rho g\cos\alpha \qquad (7)$$

$$\delta PrR\left[(u+1)\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right]=-\left[\frac{\partial^2\theta}{\partial x^2}\delta^2+\frac{\partial^2\theta}{\partial y^2}\right]+2\delta^2 N\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]+N\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2 \qquad (8)$$

where
$$S_{xx}=\frac{2\delta}{1+\lambda_1}\left[1+\frac{\delta\lambda_2 c}{a}(u\frac{\partial}{\partial x}+\frac{v}{\delta}\frac{\partial}{\partial y})\right]\frac{\partial u}{\partial x},$$

$$S_{yy}=\frac{-2\delta}{1+\lambda_1}\left[1+\frac{\delta\lambda_2 c}{a}(u\frac{\partial}{\partial x}+\frac{v}{\delta}\frac{\partial}{\partial y})\right]\frac{\partial u}{\partial y},$$

$$S_{xy}=\frac{1}{1+\lambda_1}\left[1+\frac{\delta\lambda_2 c}{a}\left(u\frac{\partial}{\partial x}+\frac{v}{\delta}\frac{\partial}{\partial y}\right)\right]\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right),$$ and $$\left(\frac{\partial S_{xy}}{\partial y}\right)_{\delta\to 0}=\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}$$

The non dimensional boundary conditions are

$$\frac{\partial u}{\partial y}=0\ ;\ \frac{\partial \theta}{\partial y}=0 \quad at\ \ y=0 \qquad (9)$$

$$u=-1\ ;\theta=1 \quad at\ \ y=h \qquad (10)$$

Using long wavelength approximation and dropping terms of order δ and higher.
Eqs. (6) - (8) reduces to

$$\frac{\partial}{\partial y}\left(\frac{1}{1+\lambda}\frac{\partial u}{\partial y}\right)-M^2(u+1)+GSin\alpha=\frac{\partial p}{\partial x} \qquad (11)$$

$$\frac{\partial p}{\partial y}=0 \qquad (12)$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2=0 \qquad (13)$$

where $M^2=\dfrac{\sigma B_0 a^2}{\mu}$

3. SOLUTION OF THE FROBLEM

Solving the Eq. (11) with the boundary conditions (9), we get

$$u=\frac{P-Gsin\alpha}{M^2cosh(Mh\sqrt{1+\lambda})}cosh((M\sqrt{1+\lambda})y)+\left(\frac{-P+Gsin\alpha-M^2}{M^2}\right) \qquad (14)$$

where $P=\dfrac{\partial p}{\partial x}$.

Integrating the equation (14) and using the conditions $\psi=0$ at $y=0$, we get the stream function as

$$\psi=\frac{1}{\mathrm{M}^2}\{\left(y\cosh[hM\sqrt{1+\lambda}]\right)(M^2+P-G\sin\alpha)+\left(\frac{(-P+G\sin\alpha)\sinh hM\sqrt{1+\lambda}}{M\sqrt{1+\lambda}}\right)* \qquad (15)$$
$$(\cosh hM\sqrt{1+\lambda}+\sinh h\sqrt{M^2(1+\lambda)})\left(-1+Tanh\sqrt{M^2(1+\lambda)}\right)\}$$

Substituting Eq. (14) into Eq. (13) subject to the boundary conditions (10), the temperature is

$$\theta = \frac{1}{8M^4}\{2M^2(2M^2 - Br(p - G\sin\alpha)^2(h^2 - y^2)(1+\lambda)) + (4M^4 + Br(p - G\sin\alpha)^2) + \cosh(2hM\sqrt{1+\lambda}) - Br(p - G\sin\alpha)^2\cosh(2My\sqrt{1+\lambda})\,\mathrm{sech}(Mh\sqrt{1+\lambda})^2\} \quad (16)$$

where $Br = EcPr$ is the Brinkman number.

The coefficient of heat transfer is given by

$$Z = h_x\theta_y. \quad (17)$$

The volume flux '*q*' through each cross section in the wave frame is given by

$$q = \int_0^h u\,dy \quad (18)$$

$$= \frac{1}{M^2}\frac{(P - Gsin\alpha)Tanh(M\sqrt{1+\lambda})h}{M\sqrt{1+\lambda}} - \left(P - Gsin\alpha + M^2\right)h \quad (19)$$

The expression for pressure gradient from Eq. (19) is given by

$$\frac{\partial p}{\partial x} = \frac{M\sqrt{1+\lambda}((q)M^2 + (M^2 - Gsin\alpha)h) + Gsin\alpha.Tanh(M\sqrt{1+\lambda}\,h)}{Tanh(M\sqrt{1+\lambda}\,h) - Mh\sqrt{1+\lambda}} \quad (20)$$

The instantaneous volume flow rate $Q(x,t)$ in the laboratory frame between the central line and the wall is

$$Q(x,t) = \int_0^h (u+1)dy = q + h \quad (21)$$

Averaging the Eq. (21) over one period yields the time mean flow rate (time averaged flow rate) $\bar{Q}$ as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\,dt = q + 1 \quad (22)$$

The pressure difference and frictional force across one wave length in an inclined channel is given by

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x}dx \quad (23)$$

$$F = \int_0^1 h - \frac{\partial p}{\partial x}dx \quad (24)$$

4. Results and Discussion

To Study the behavior of the distribution of the axial velocity *u*, numerical calculations for several values of Jeffrey fluid parameter λ, Hartman number M, angle of inclination α, gravitational parameter *G* are carried out. Fig. 2(a) shows that an increase in λ results in increase of velocity distribution. The effect of α on the velocity distribution can be seen through Fig. 2(b). It reveals that the axial velocity decreases with increasing α. Fig. 2(c) displays the effect of G on the velocity distribution for fixed values of other parameters. It is observed that the velocity increases with increase of *G*. The effect of the *M* on the velocity distribution is illustrated in Fig. 2(d). It is evident that, increase the value of *M* has a tendency to slow down the fluid motion and fluid moves like a block, which shows some sort of rigidity. This is because of the presence of the transverse magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid motion, thus causing the velocity of the fluid to decrease.

The effect of heat transfer on peristalsis is illustrated in Fig.3. Fig. 3(a) is made to see the variation temperature θ for various values of Brinkman number Br. It is observed that the temperature profiles are almost parabolic and the increase of Br, the temperature distribution increases. From Fig. 3(b), it can be noticed that the temperature decreases with an increase of *M*. Figs. 3(c) and 3(d) are plotted to see the influence of Jeffrey fluid parameter and angle of inclination on the temperature distribution. It is observe that an increase in Jeffrey fluid parameter λ affects the temperature profile in an opposite way to that of angle of inclination.

Fig. 4 is plotted to study the effect of Jeffrey fluid parameter λ, Hartman number M, angle of inclination α, gravitational parameter *G* on the pressure gradient Figs. 4(a) and 4(c) indicate that the pressure gradient increases with increase of *M* and *G*. From Figs. 4(b) and 4(d), it is observe that pressure gradient decreases with increase of α and λ.

Fig.5 shows the behavior of heat transfer co-efficient *Z*. From this figure it is observed that due to peristalsis the heat transfer coefficient is in oscillatory behavior. The absolute value of heat transfer coefficient decreases with increasing α and M while it increases with increase of Br and λ.

5. TRAPPING

Trapping is an interesting phenomenon in peristaltic motion. It is basically the formation of an internally circulating bolus of the fluid by closed stream lines. The effect of *M* on trapping can be seen in Fig. 6. We observed that the size of the bolus reduces with an increase in *M*. The effects of λ and *G* on the stream lines are plotted in Figs.7-8. It is observed that the size of trapping bolus increases with increasing λ and *G*.

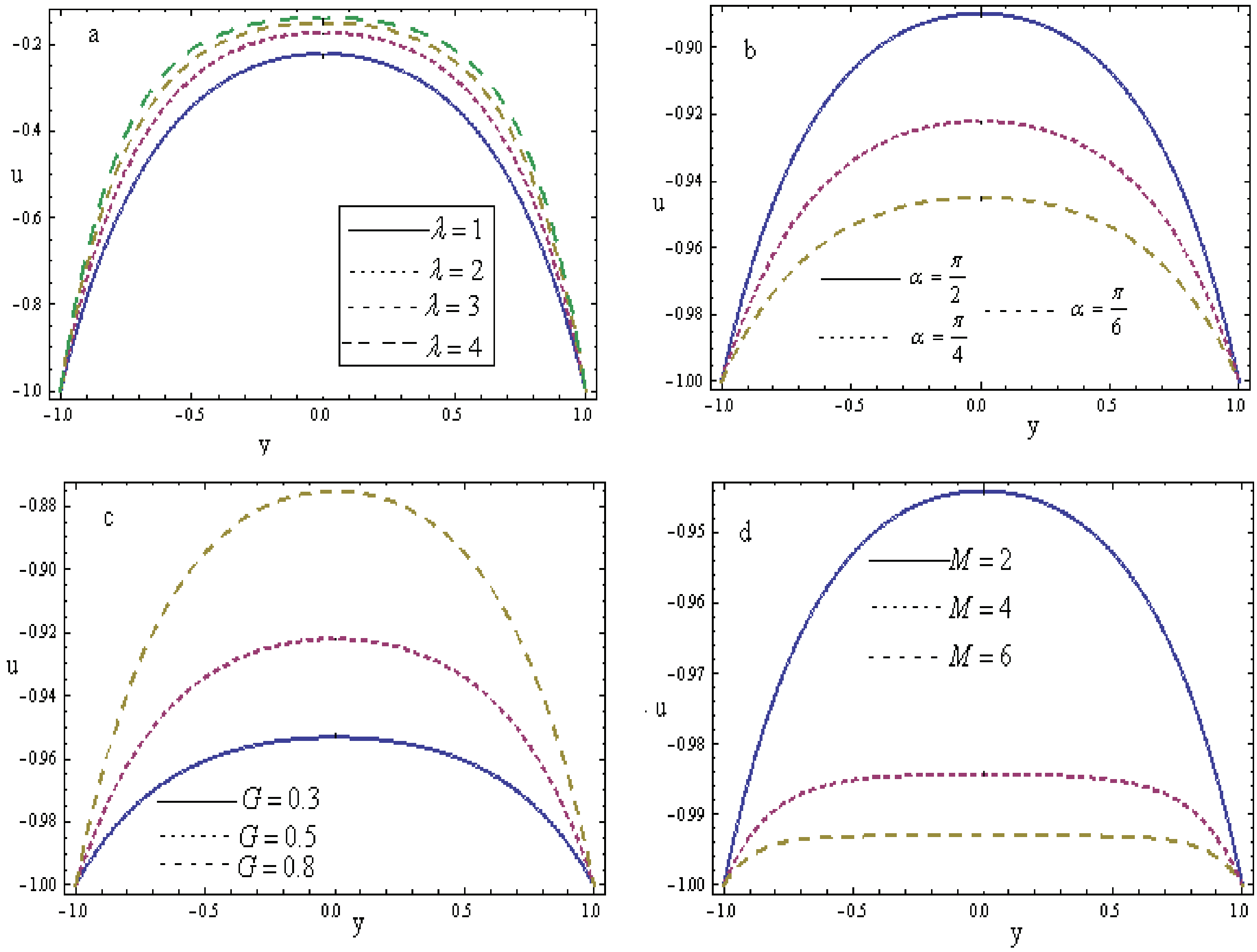

Figure. 2. The velocity distribution for ($\phi = 0.4$, $x = 0.25$); other parameters are (a) $\alpha = \frac{\pi}{4}, G = 0.5, M = 2$, (b) $M = 2, G = 0.5, \lambda = 1$, (c) $\alpha = \frac{\pi}{4}, M = 2, \lambda = 1$, (d) $\alpha = \frac{\pi}{4}, G = 0.5, \lambda = 1$.

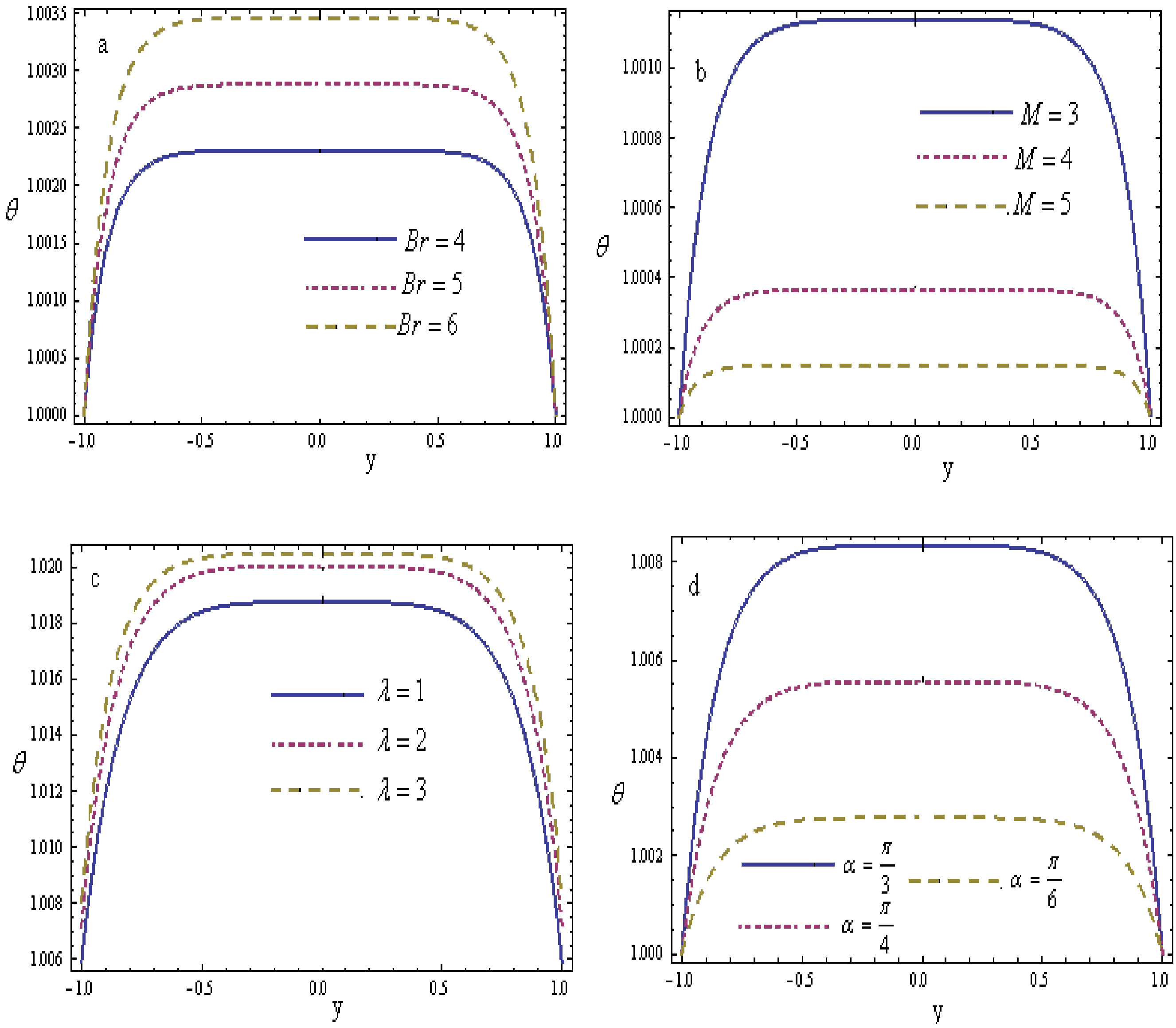

Figure. 3.Temperature distribution for ($\phi = 0.4$, $x = 0.25$); other parameters are

(a) $\alpha = \frac{\pi}{4},\ G = 0.5,\ M = 2,\ \lambda = 1$. (b) $\alpha = \frac{\pi}{4},\ G = 0.5,\ \lambda = 1,\ Br = 3$.

(c) $\alpha = \frac{\pi}{4},\ M = 2,\ G = 0.5,\ Br = 3$. (d) $M = 2,\ G = 0.5,\ \lambda = 1,\ Br = 3$.

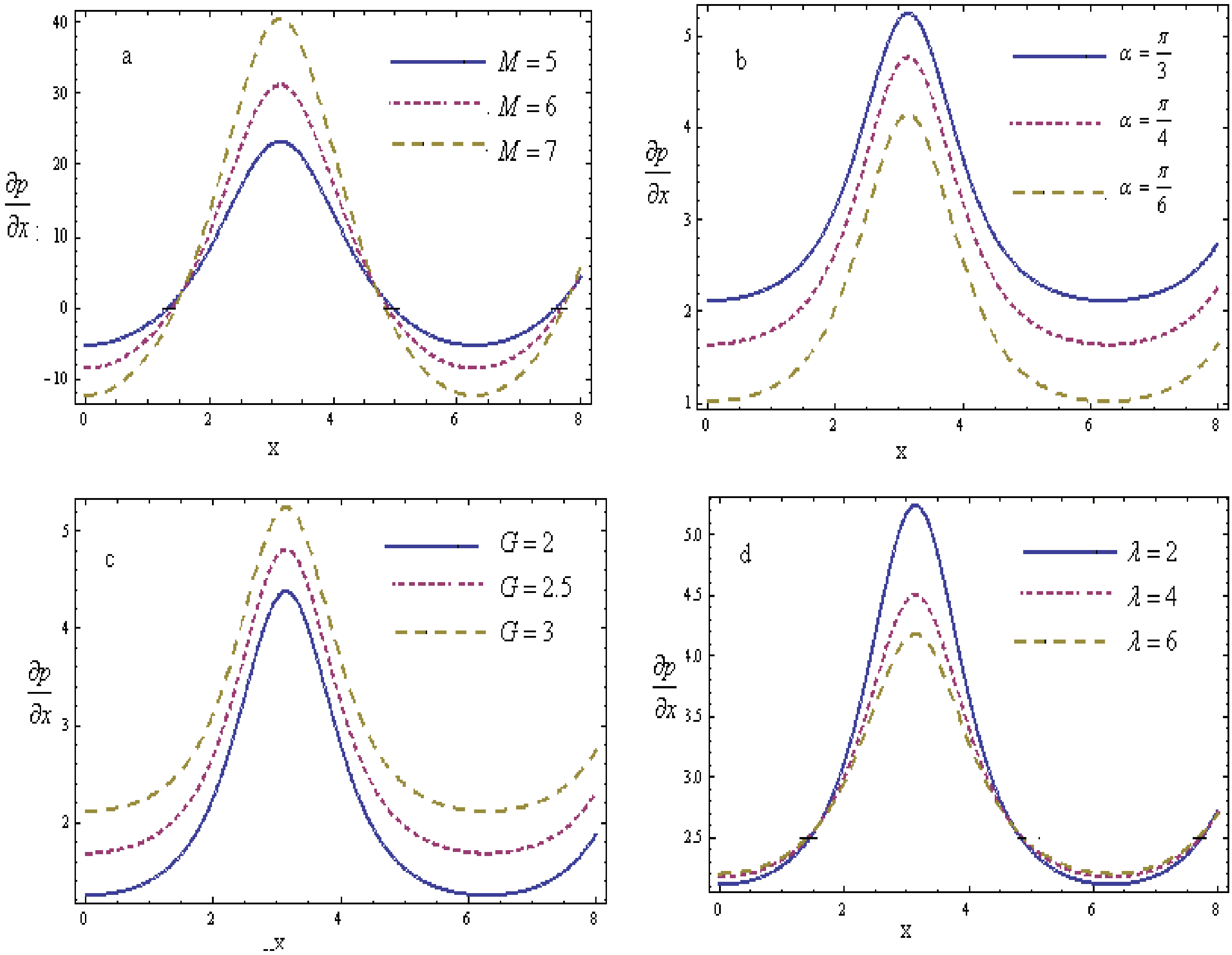

Figure. 4. Pressure gradient versus x for (a) $\phi = 0.4$, $\lambda = 2, \alpha = \dfrac{\pi}{3}, G = 3$.

(b) $G = 2, \lambda = 2, M = 1, \phi = 0.4$. (c) $\alpha = \dfrac{\pi}{3}, \lambda = 2, M = 1, \phi = 0.4$. (d) $\alpha = \dfrac{\pi}{3}, G = 2, M = 1, \phi = 0.4$.

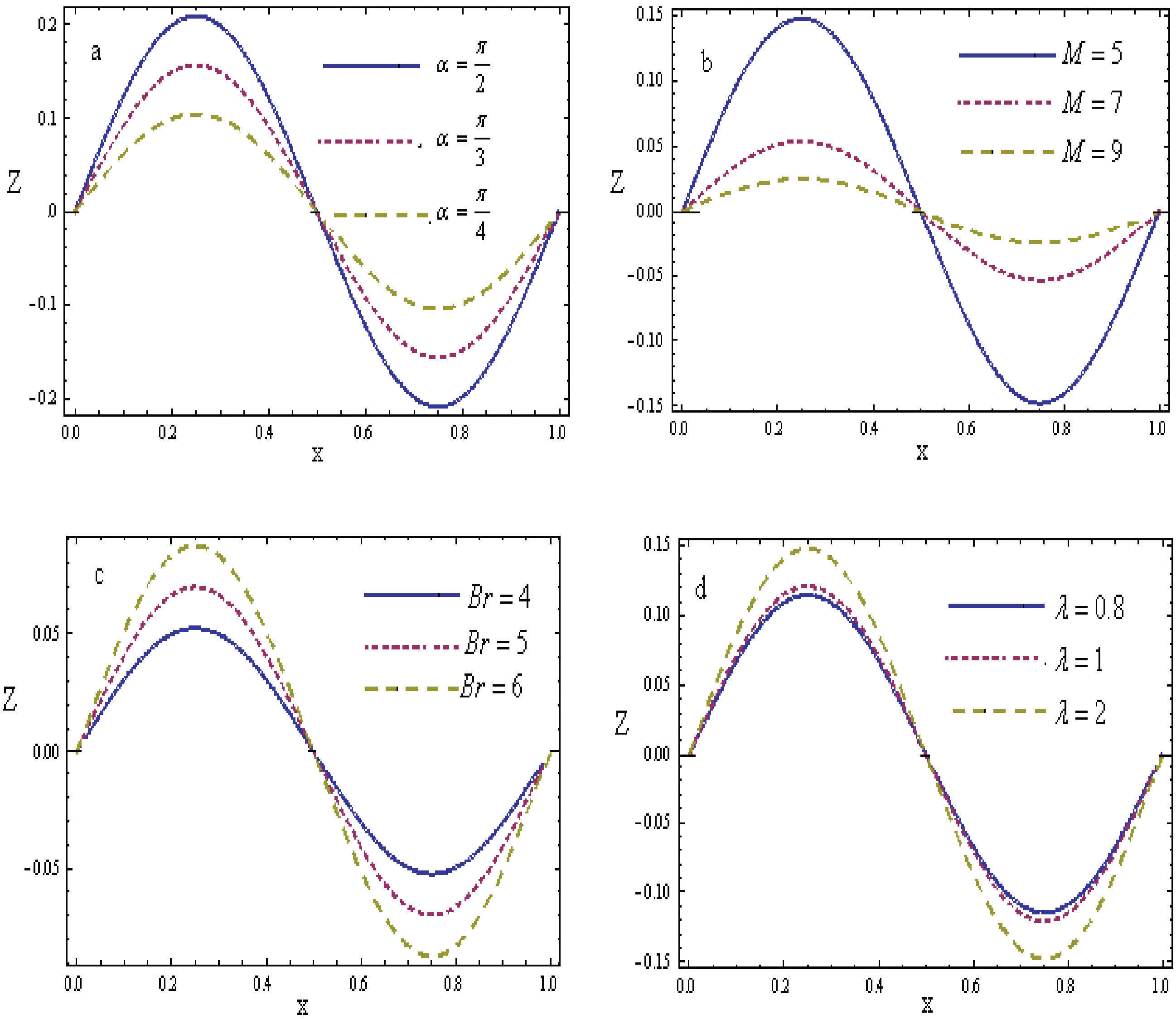

Figure. 5. Coefficient of heat transfer for (a) $\phi = 0.6,\ M = 5,\ G = 0.2,\ \lambda = 2,\ Br = 3.$

(b) $\alpha = \frac{\pi}{3},\ G = 0.2,\ \lambda = 2,\ Br = 3,\ \phi = 0.6$. (c) $\alpha = \frac{\pi}{3},\ G = 0.2,\ \lambda = 2,\ M = 5,\ \phi = 0.6$

(d) $\alpha = \frac{\pi}{3},\ G = 0.2,\ Br = 3,\ M = 5,\ \phi = 0.6$.

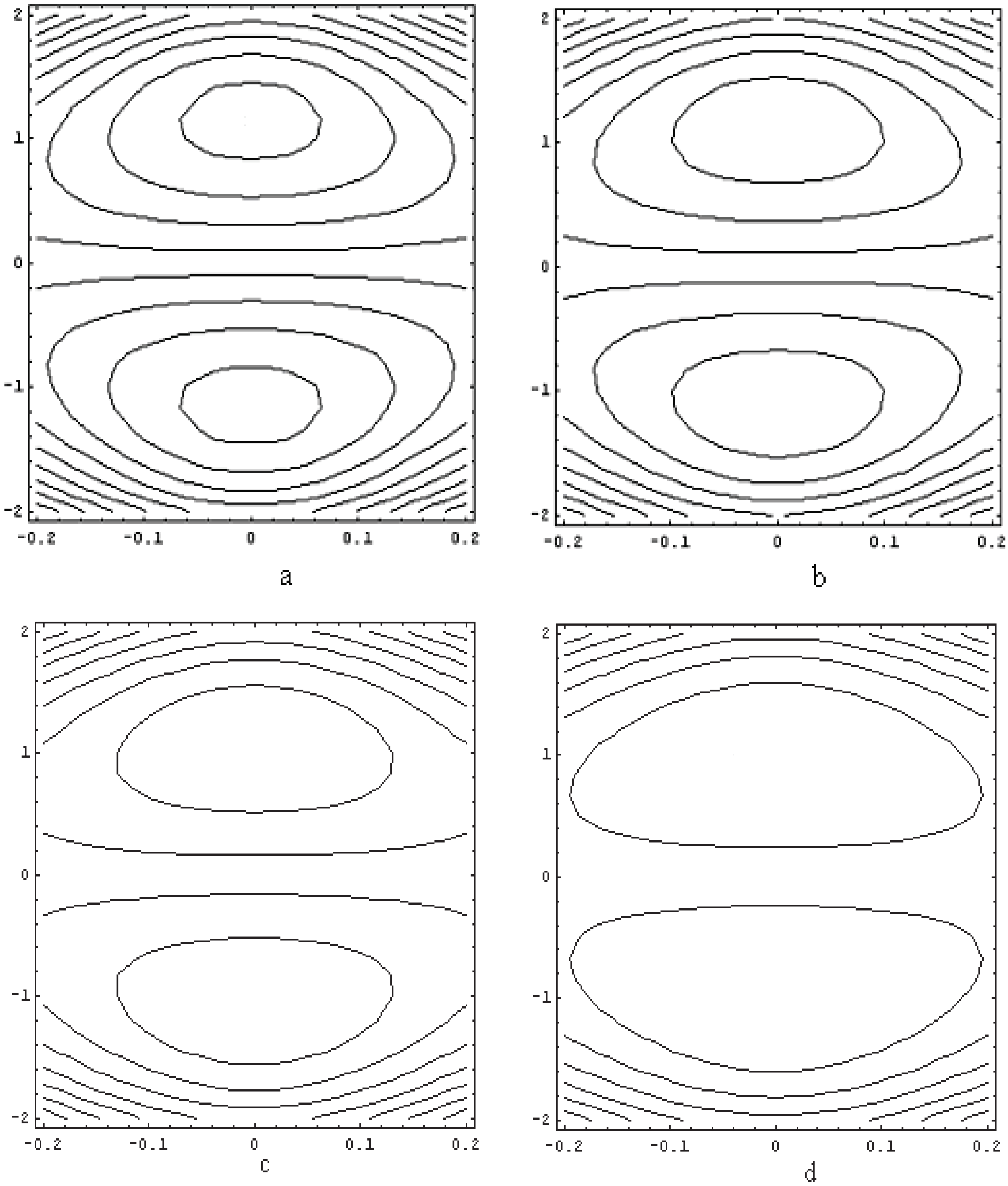

Figure. 6. Streamlines for (a) $M = 0.1$. (b) $M = 0.2$. (c) $M = 0.3$. (d) $M = 0.4$; other parameters are $\phi = 0.4, \theta = 30^0, \lambda = 4, G = 0.8$.

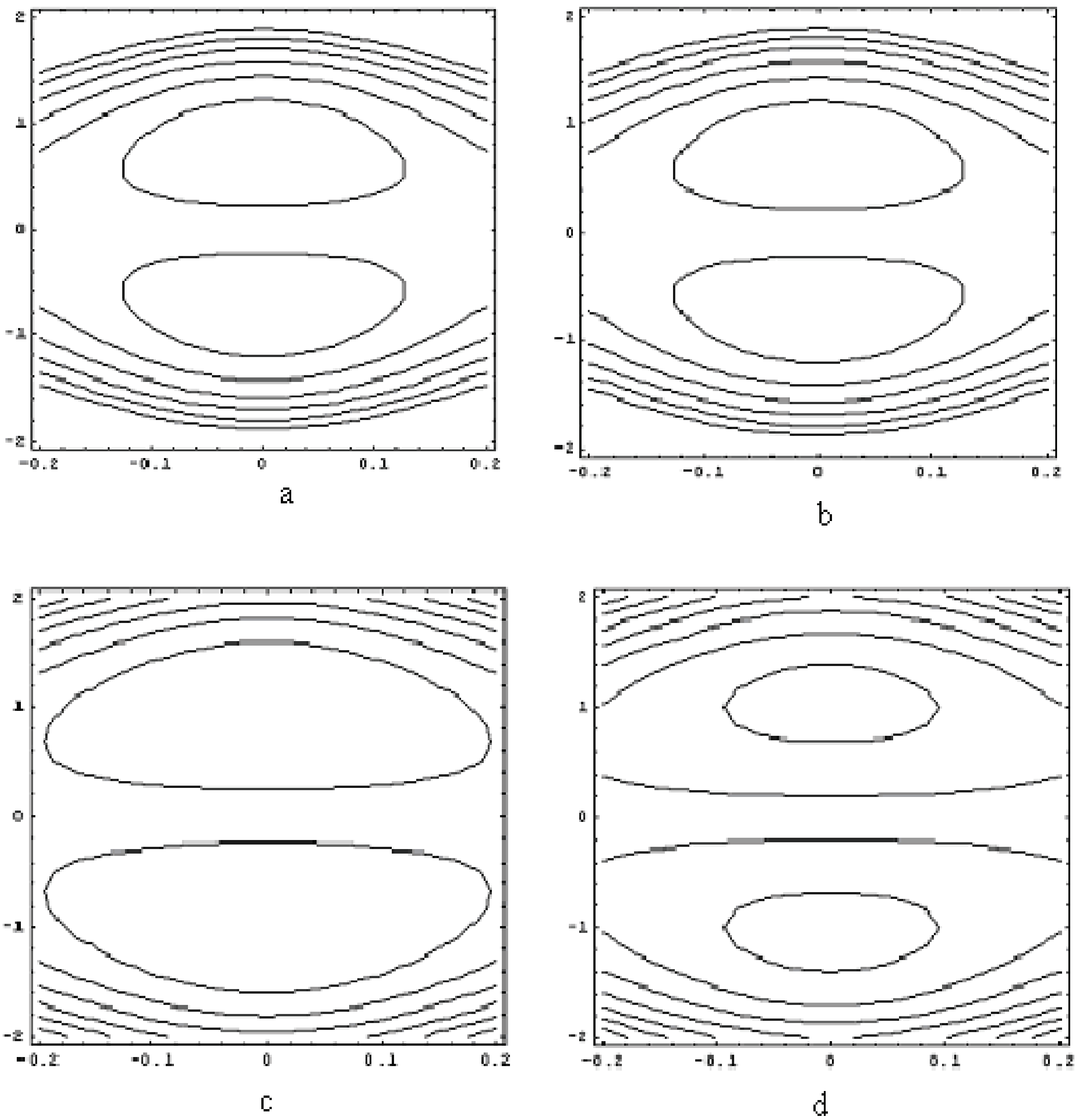

Figure. 7. Streamlines for (a) $\lambda = 2$ (b) $\lambda = 3$ (c) $\lambda = 4$ (d) $\lambda = 5$; other parameters are $\phi = 0.4, \theta = 30^0, M = 0.4, G = 0.8$

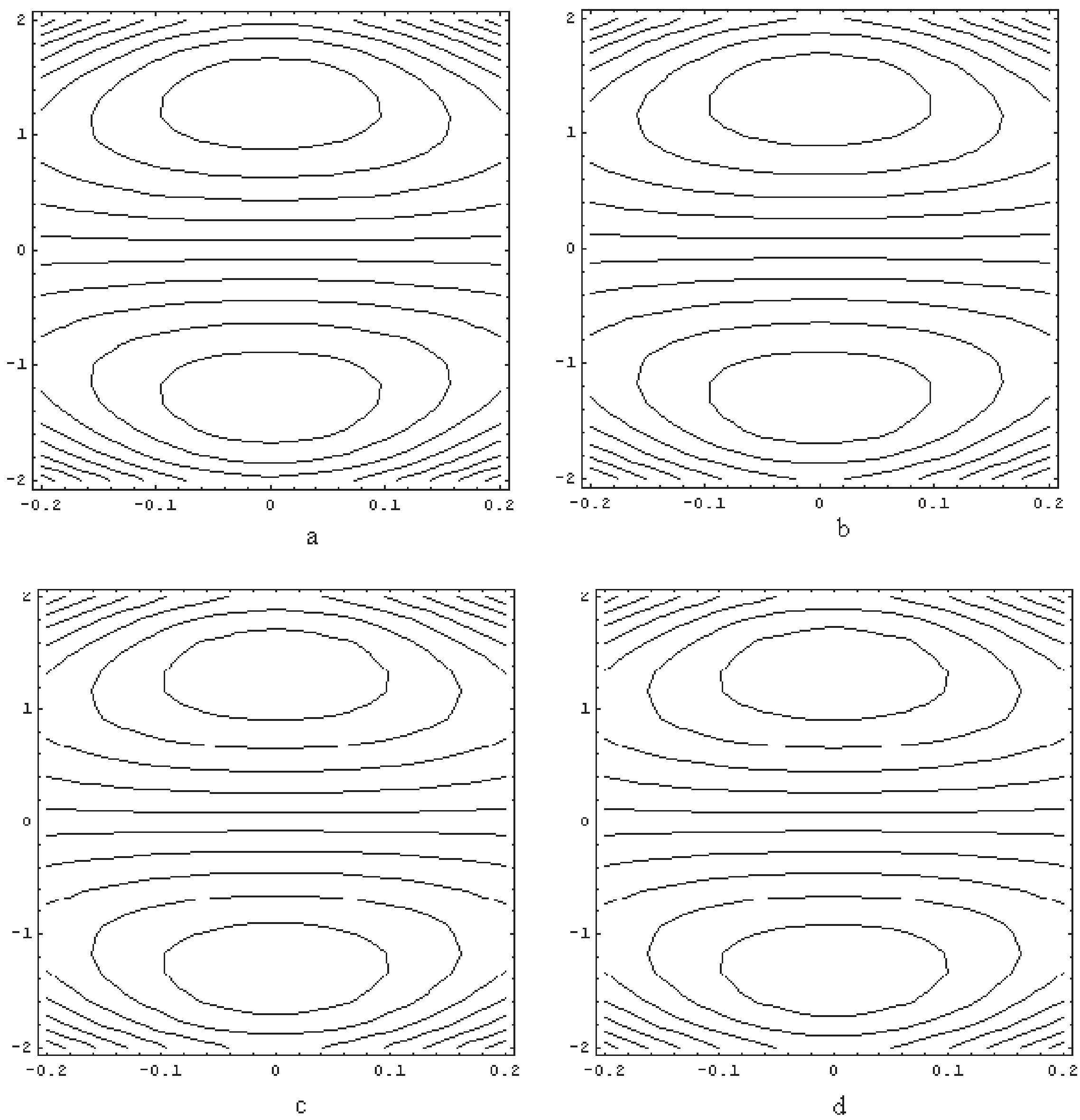

Figure.8. Streamlines for different values of (a) $G = 4$ (b) $G = 5$ (c) $G = 6$ (d) $G = 7$; Other parameters are $\phi = 0.4, \theta = 30^0, M = 0.4, \lambda = 2$.

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