# Dirichlet series and approximate method for the solution of axisymmetric flow over a stretching sheet

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***Abstract***— We study the boundary layer flows induced by the axisymmetric stretching of a sheets are studied using more suggestive schemes. The equation of motion of a axisymmetric flow over a stretching sheet and the sheet is stretched with a velocity is proportional to the distance from the vertical axis. The governing nonlinear differential equations are reduced to nonlinear linear ordinary differential equations (ODEs) by using similarity transformations. The resulting nonlinear ODEs are solved by using fast convergent Dirichlet series method and an approximate analytical method by Method of stretching of variables. These methods have advantages over pure numerical methods for obtaining the derived quantities accurately for various values of the parameters involved at a stretch and these are valid in much larger domain as compared with the classical numerical schemes.

**Keywords: Axisymmetric flow; boundary layer equations; Stretching sheet; Dirichlet series; Powell's method; Least square approximation**

## I. INTRODUCTION

In this discussion, we consider boundary layer flow over axisymmetric stretching of a sheet which are of significant interest in the recent years due to their applications. The third order nonlinear ordinary differential equation over an infinite interval with parameter *M*, Hartman number (magnetic field) is of special interest and in very few specific cases they have analytical solutions. The flow of a viscoelastic fluid over a stretching sheet was investigated by Rajagopal et al. [1], Sarpkaya [2] who probably the first to consider the MHD flow of non-Newtonian fluids. Andersson [3] and Mamaloukas et al. [4] have obtained similarity solution of the boundary layer equation governing the flow of a viscoelastic and a second grade fluid past a stretching sheet in the presence of an external magnetic field. The fluid occupies the space above the sheet and the motion is caused by stretching sheet in opposite directions with the velocity is proportional to the distance from the fixed axis studied by Crane [5], and also he has given an elegant solution of the problem . The more interesting fact is that the problem still admits an exact analytical solution. The other effects are taken into account, such as suction at the sheet was discussed by (Gupta and Gupta [6]), viscoelasticity of the fluid by Ariel [7,8], partial slip at the boundary by Wang [9]. The problem of flow due to the radial stretching of the sheet (i. e the velocity of the sheet is proportional to the distance from a vertical rather than a horizontal axis) does not have an exact solution. For this reason, this problem has received much less attention in the literature. Wang [10], discussed the numerical solution of the flow due to the radial stretching of the sheet. The effects of visco-elasticity were studied by Ariel for an elastico-viscous fluid [11] and the second grade fluid by [12]. Areal [13], discussed the axisymmetric flow due to stretching of a sheet in hydromagnetics as the prototype problem for the non-iterative algorithm and also develops an algorithm for solving the problems of the flow induced by the moving boundaries in hydromagnetics. Hayat et al.[14] has used modified decomposition method and Pade' approximants, for the solution of equation third order nonlinear ODE with infinite interval arsing in MHD. Shahzad et al [15], investigated the exact solution for axisymmetric flow and heat transfer over nonlinearly radially stretching sheet by HAM. Recently, Khan and Shahzad [16] have analysed the axisymmetric flow of sisko fluid over a radially stretching sheet using HAM.

The present investigation is to analyze the boundary layer flow induced by axisymmetric stretching of a sheet given by Mirgolbabaei et al [17]. The solution of the resulting third order nonlinear boundary value problem with infinite interval is obtained by Dirichlet series method and approximation method. We seek solution of the general equation of the type

$$f''' + Aff'' + Bf'^2 + Cf' = 0 \quad (1)$$

with the boundary conditions

$$f(0) = \alpha_1,\ f'(0) = \beta_1,\ f'(\infty) = 0) \quad (2)$$

where *A, B* and *C* are constants and prime denotes derivative with respect to the independent variable $\eta$. This equation admits a Dirichlet series solution; necessary conditions for the existence and uniqueness of these solutions may also be found in [18, 19]. For a specific type of boundary condition i.e. $f'(\infty) = 0$, the Dirichlet series solution is particularly useful for obtaining the derived quantities. A general discussion of the convergence of the Dirichlet series may also be found in Riesz [20]. The accuracy as well as uniqueness of the solution can be confirmed using other powerful semi-numerical schemes. Sachdev et al. [21] have analyzed various problems from fluid dynamics of stretching sheet using this approach and found more accurate solution compared with earlier numerical findings. Recently, Awati et al [22, 23] and Kudenatii et al [24] have analysed the problems from MHD boundary layer flow with nonlinear stretching sheet using the above methods and found more accurate results compared with the classical numerical methods. Dirichlet series solution and MSV which we present here is more attractive than adapted variational iteration method (AVIM) discussed by Mirgolbabaei et al [17].

The present work is structured as follows. In section 2 the mathematical formulation of the proposed problem with relevant boundary conditions is given. Section 3 is devoted to semi-numerical method for the solution of the problem using Dirichlet series. In section 4 the solution of the proposed problem by an approximate analytical method using the method of stretching of variables (MSV). In section 5 detailed results obtained by the novel method explained here are compared with the corresponding numerical schemes. Section 6 Conclusions.

## II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the equation of motion of electrically conducting, viscous incompressible fluid caused by radial stretching sheet at *z=0* in the presence of transverse magnetic field. The stretching velocity of the sheet is proportional to the distance from the origin of the sheet. In the cylindrical polar coordinates $(r, \theta, z)$ and the flow takes place in the upper half plane $z > 0$. In view of the rotational symmetry of the flow all physical quantities are independent of $\theta$ i.e $\partial/\partial\theta \equiv 0$. The equation of motion for steady, laminar, axisymmetric flow and continuity are of the form (Mirgolbabaei et al [17])

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right) - \frac{\sigma B_0}{\rho}u, \quad (3)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right), \quad (4)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (5)$$

where $\rho$ is fluid density, $\mu$ is coefficient of viscosity, $\sigma$ is the electrical conductivity of the fluid, *p* is the pressure and $(u, 0, w)$ are the velocity components along $(r, \theta, z)$ directions. The boundary conditions for the above flow situations are

$$\left.\begin{array}{l} u = cr, \quad w = 0 \ \text{at} \ z = 0 \\ u \to 0 \ \text{as} \ z \to \infty \end{array}\right\} \quad (6)$$

where *c* > 0 is the constant of proportionality relating to the stretching of the sheet. The boundary layer Eq. (3)-

.(6) admit the similarity solution (Wang [9]),

$$u = crf'(\eta),\ w = -2\sqrt{c\upsilon}\, f(\eta),\ \eta = \sqrt{(c/\upsilon)}\, z \qquad (7)$$

where $\upsilon = \mu/\rho$ is the kinematic viscosity of the fluid and prime denotes differentiation with respect to $\eta$, eventually reduce the Navier-Stokes equation to an ODE. From Eq. (3), we get

$$\frac{\partial p}{\partial r} = \rho c^2 r\left[f''' + 2ff'' - f'^2 - Mf'\right] \qquad (8)$$

where $M = \sigma B_0/\rho c$ is the magnetic parameter. On the other hand Eq. (4), gives

$$\frac{\partial p}{\partial \eta} = -4c\mu ff' - 2c\mu f'' \qquad (9)$$

which when integrated with respect to $\eta$ yields,

$$p = -2c\mu f^2 - 2c\mu f' + g(r), \qquad (10)$$

where *g(r)* is an arbitrary function of *r*. Substitute for *p* Eq. (10) into Eq. (8), we obtain

$$\frac{g'(r)}{\rho c^2 r} = f''' + 2ff'' - f'^2 - Mf' \qquad (11)$$

Since in Eq. (11), the left hand side is a function of $r$ only, and the right hand side is a function of $\eta$ only, in order for it to be consistent, each side must be constant, say $A_1$. Hence we have

$$g'(r) = \rho c^2 r A_1 \qquad (12)$$

Its integration with respect to *r*, gives

$$g(r) = p_0 + (1/2)\rho c^2 r^2 A_1 \qquad (13)$$

where $p_0$ is a constant. Substitution of *g(r)* from Eq.(13) into Eq.(10) leads to

$$p = p_0 + (1/2)\rho c^2 r^2 A_1 - 2c\mu f^2 - 2c\mu f' \qquad (14)$$

Since the entire motion of the fluid is caused due to stretching of the sheet, the pressure far away from the sheet must be given by the Bernoulli's equation, i.e., Matching of the pressure from Eq. (14), gives $A_1=0$. Hence from equation (11), we get the following DE for *f* (Mirgolbabaei et al [17])

$$f''' + 2ff'' - f'^2 - Mf' = 0 \qquad (15)$$

Also the pressure *p* at any point in terms of the physical variables is

$$p = p_0 - \frac{1}{2}\rho w^2 + \mu\frac{\partial w}{\partial z} \qquad (16)$$

The boundary conditions of the problem are

$$f(0) = 0,\quad f'(0) = 1,\quad f'(\infty) = 0 \qquad (17)$$

## III. DIRICHLET SERIES APPROACH TO THE BOUNDARY VALUE PROBLEMS OVER AN INFINITE INTERVAL

We seek a Dirichlet series solution of equation (1) satisfying the last boundary condition of Eq.(2) automatically i.e $f'(\infty) = 0$ in the form of (Kravchenko and Yablonskii [18, 19])

$$f = \gamma_1 + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i e^{-i\gamma\eta} \qquad (18)$$

where $\gamma$ and *a* are parameters. Substituting (18) into (1), we get

$$\sum_{i=1}^{\infty}\left\{-\gamma^2 i^3 + A\gamma\gamma_1 i^2 - C i\right\} b_i a^i e^{-i\gamma\eta} + \frac{6\gamma^2}{A}\sum_{i=2}^{\infty}\sum_{k=1}^{i-1}\left\{A k^2 + B k(i-k)\right\} b_k b_{i-k} a^i e^{-i\gamma\eta} = 0 \quad (19)$$

For $i=1$, we have $\gamma_1 = \frac{\gamma^2 + C}{A}$ (20)

Substituting (20) into (19) the recurrence relation for obtaining coefficients is given by

$$b_i = \frac{6\gamma^2}{A i(i-1)\left\{\gamma^2 i - C\right\}}\sum_{k=1}^{i-1}\left\{A k^2 + B k(i-k)\right\} b_k b_{i-k} \quad (21)$$

For $i = 2, 3, .....$ . If the series (18) converges absolutely when $\gamma > 0$ for some $\eta_0$, this series converges absolutely and uniformly in the half plane $\mathrm{Re}\,\eta \geq \mathrm{Re}\,\eta_0$ and represents an analytic $(2\pi i/\gamma)$ periodic function $f = f(\eta_0)$ such that $f'(\infty) = 0$ (Kravchenko & Yablonskii [19]). The series (18) contains two free parameters namely $a$ and $\gamma$. These unknown parameters are determined from the remaining boundary conditions (2) at $\eta = 0$

$$f(0) = \frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i = \alpha_1 \quad (22)$$

and $$f'(0) = \frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i) b_i a^i = \beta_1 \quad (23)$$

The solution of these transcendental equations (22) and (23) yield, constants $a$ and $\gamma$. The solution of these transcendental equations is equivalent to the unconstrained minimization of the functional

$$\left[\frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i - \alpha_1\right]^2 + \left[\frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i) b_i a^i - \beta_1\right]^2 \quad (24)$$

We use Powell's method of conjugate directions (Press et al [25]) which is one of the most efficient techniques for solving unconstrained optimization problems. This helps in finding the unknown constants $a$ and $\gamma$ uniquely for different values of the parameters $A$, $B$, $C$, $\alpha_1$ and $\beta_1$. Alternatively, Newton's method is also used to determine the unknown parameters $a$ and $\gamma$ accurately. The shear stress at the surface of the problem is given by

$$f''(0) = \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i (i\gamma)^2 \quad (25)$$

The velocity profiles of the problem is given by

$$f'(\eta) = \frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i) b_i a^i e^{-i\gamma\eta} \quad (26)$$

## IV. METHOD OF STRETCHING OF VARIABLES

Many nonlinear ODE arising in MHD problems are not amenable for obtaining analytical solutions. In such situations, attempts have been made to develop approximate methods for the solution of these problems. The numerical approach is always based on the idea of stretching of variables of the flow problems. Method of stretching of variables is used here for the solution of such problems. In this method, we have to choose suitable derivative function $H'$ such that the derivative boundary conditions are satisfied automatically and integration of $H'$ will satisfy the remaining boundary condition. Substitution of this resulting function into the given equation gives the residual of the form $R(\xi, \alpha)$ which is called defect function. Using Least squares method, the residual of the defect function can be minimized. For details see (Ariel, [26]). Using the transformation $f = f_w + F$ into Eq. (1), we get

$$F''' + A(f_w + F)F'' + BF'^2 + CF' = 0, \quad 27)$$

and the boundary conditions (2) become

$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = 0 \quad (28)$$

We introduce two variables $\xi$ and $G$ in the form

$G(\xi) = \alpha F(\eta)$ and $\xi = \alpha\eta$ (29)

where $\alpha$ >0, is an amplification factor. In view of Eq.(29), the system (27-28) are transformed to the form

$$\alpha^2 G''' + A(f_w \alpha + G)G'' + BG'^2 + CG' = 0, \quad (30)$$

and the boundary conditions in Eq. (28) become

$$G(0) = 0, \quad G'(0) = 1, \quad G'(\infty) = 0 \quad (31)$$

We choose a trail velocity profile

$$G' = \exp(-\xi) \quad (32)$$

which automatically satisfies the derivative conditions in Eq.(31). Integrating Eq. (32) with respect to $\xi$ from 0 to $\xi$ using the first boundary conditions in (31) and then substituting this into Eq. (30), we get the residual of the defect function

$$R(\xi, \alpha) = (\alpha^2 - Af_w\alpha - A + C)\exp(-\xi) + (A + B)\exp(-2\xi) \quad (33)$$

By using the least squares method as discussed in Ariel [26], the equation (33) can be minimized for which

$$\frac{\partial}{\partial \alpha}\int_0^\infty R^2(\xi, \alpha)d\xi = 0. \quad (34)$$

Substituting (33) into equation (34) and solving cubic equation in $\alpha$ for a positive root, we get

$$\alpha = \frac{1}{6}\left(3Af_w \pm \sqrt{3}\sqrt{4A - 8B - 12C + 3A^2 f_w^2}\right)$$

and $\alpha = \frac{Af_w}{2}$.

(35)

Once the amplification factor is calculated, then using Eq.(27), original function *f* can be written as

$$f = f_w + \frac{1}{\alpha}(1 - \exp(-\alpha\eta)). \quad (36)$$

with $\alpha$ defined in Eq. (35). Thus Eq. (36) gives the solution of Eq. (1) for all $A$, *B*, *C and* $f_w$.

## V. RESULT AND DISCUSSION

In the present paper, the axisymmetric flow over a stretching of a sheet is discussed by using semi-numerical method and approximate analytical method. The Eq. (15) and (17) are solved semi-numerically using one of the powerful techniques due to Dirichlet series method and the method of stretching of variables. We have given an exact analytical solution of the boundary value problem in more general form. In this semi-numerical method and, it is important to note that the edge boundary condition automatically satisfied an also we have given analytical solution by approximate method.

**Case I.** Consider the flow of a fluid with no magnetic field Eq. (15) reduces to $f''' + 2ff'' - f'^2 = 0$ and the boundary conditions are same as of Eq.(17). We check the validity of our solution by comparing it with the exact solution. The measure of the physical quantity viz. shear stress at the sheet is $-f''(0)$. The exact value of $-f''(0)$ is 1.173721 and the value obtained by Dirichlet series is 1.173721 and MSV is 1.15470.The error being very less as compared to AVIM.

**Case II.** In this case, consider the flow of an electrically conducting, viscous incompressible fluid over a radially stretching sheet in presence of a transverse magnetic filed. The governing equation is same as Eq. (15) and $\sqrt{M}$ is the Hartmann number. As the value of *M* increased, Hartmann layers start in at $\eta = 0$ causing the great difficulties in obtaining the numerical solution. We have presented much better solution than AVIM by using Dirichlet series and MSV for arbitrary values of *M* which are comparable with exact numerical solutions by Ariel [26] which are listed in Table 1. The above said methods are capable for providing  the solutions in the presence of Hartmann layers near the stretching sheet.

**Case III.** The massive transfer of the fluid across the boundary, the type of the boundary layer is manifested near the boundary. The semi-numerical and approximation method are able to handle the suction boundary layer in an efficient manner with Hartmann layer. The suction takes place across the sheet, the boundary conditions (17) changes to $f(0)=f_w, f'(0)=1, f'(\infty)=0$, where $f_w$ is the suction parameter given by $f_w=(w_0/2)\sqrt{(\rho/c\mu)}$ , where $w_0$ is the suction velocity. The governing equation for *f* is same as Eq. (15). The values of $-f''(0)$ are presented for various a vales of $f_w$ using Dirichlet series method and MSV are given in Table 2. and these values are comparable with numerical solution given by Ackroyd [27].

**Case IV**. In this case, we consider the flow of an electrically conducting viscous incompressible fluid due to radial stretching of a sheet in the presence of the transverse magnetic field and also the suction at the sheet. The governing equation is same as Eq. (15) and relevant boundary conditions $f(0)=f_w, f'(0)=1, f'(\infty)=0$. Numerical computations are performed by using the above said methods for various values of the physical parameters involved in the equation viz., Hortmann number *M*, and mass suction parameter $f_w$ . The present solutions are then validated by comparing it with the previously published work of Migolbabaei et al [17] as shown in the Tables 3.

**Table1.** Comparison of Dirichlet series method, Method of stretching of variables (MSV) with exact solution and Adapted variational Iteration method (AVIM) for the flow in the presence of magnetic field.

| M | Dirichlet Series Method | | | Exact | MSV | AVIM |
|---|---|---|---|---|---|---|
| | $a$ | $\gamma$ | $-f''(0)$ | $-f''(0)$ | $-f''(0)$ | $-f''(0)$ |
| **0** | -0.56719 | 1.50299 | 1.17372 | 1.17372 | 1.15470 | 1.182125 |
| **0.01** | -0.18857 | 1.50659 | 1.17606 | 1.17783 | 1.15902 | 1.192910 |
| **0.04** | -0.18627 | 1.51758 | 1.17902 | 1.17901 | 1.17189 | 1.209962 |
| **0.25** | -0.16798 | 1.57558 | 1.25906 | 1.27303 | 1.25831 | 1.298851 |
| **1.0** | -0.12258 | 1.77491 | 1.53417 | 1.53571 | 1.52752 | 1.556834 |
| **4.0** | -0.05861 | 2.46267 | 2.31048 | 2.31172 | 2.30940 | 2.322880 |
| **25.0** | -0.01249 | 5.19738 | 5.13178 | 5.13181 | 5.13160 | 5.151863 |
| **100** | -0.00328 | 10.09967 | 10.06632 | 10.06647 | 10.06645 | 10.087672 |
| **500** | -0.00066 | 22.40537 | 22.39211 | --- | 22.39047 | |
| **1000** | -0.00033 | 31.65439 | 31.64398 | --- | 31.64385 | |

**Table 2.**Comparison of Dirichlet series method, Method of stretching of variables (MSV) with exact solution and Adapted variational Iteration method (AVIM) for the flow in the presence of magnetic field.

| *A* | Dirichlet Series Method | | | Exact | MSV | AVIM |
|---|---|---|---|---|---|---|
| | $a$ | $\gamma$ | $-f''(0)$ | $-f''(0)$ | $-f''(0)$ | $-f''(0)$ |
| **0** | -0.56719 | 1.50299 | 1.17372 | 1.17372 | 1.15470 | 1.182125 |
| **0.1** | -0.16335 | 1.59707 | 1.27865 | 1.28242 | 1.25902 | 1.329105 |
| **0.2** | -0.14159 | 1.69558 | 1.41216 | 1.40236 | 1.37189 | 1.488416 |
| **0.5** | -0.09085 | 2.04339 | 1.79567 | 1.79867 | 1.75931 | 2.057265 |

**Table 3.** Comparison of Dirichlet series method, Method of stretching of variables(MSV) with exact solutionand Adapted variational Iteration method (AVIM) for MHD flow with sucion.

| M | *A* | Dirichlet Series Method | | | Exact | MSV | AVIM |
|---|---|---|---|---|---|---|---|
| | | $a$ | $\gamma$ | $-f''(0)$ | $-f''(0)$ | $-f''(0)$ | $-f''(0)$ |
| **1** | 0 | -0.12258 | 1.77491 | 1.53417 | 1.53571 | 1.52753 | 1.556834 |
| | 0.1 | -0.10843 | 1.87345 | 1.64267 | 1.64312 | 1.63079 | 1.683059 |
| | 0.2 | -0.09608 | 1.97844 | 1.75491 | 1.75664 | 1.74056 | 1.821750 |
| | 0.5 | -0.06724 | 2.32755 | 2.13048 | 2.13194 | 2.10728 | 2.328433 |
| **4** | 0 | -0.05861 | 2.46267 | 2.31049 | 2.31172 | 2.30940 | 2.322880 |
| | 0.1 | -0.05386 | 2.56390 | 2.41458 | 2.41586 | 2.41156 | 2.444133 |
| | 0.2 | -0.04948 | 2.66927 | 2.52468 | 2.52422 | 2.51804 | 2.576313 |
| | 0.5 | -0.03855 | 3.00932 | 2.87325 | 2.87403 | 2.86291 | 3.051377 |
| **25** | 0 | -0.01249 | 5.19738 | 5.13178 | 5.13181 | 5.13160 | 5.15186 |
| | 0.1 | -0.01202 | 5.29831 | 5.23308 | 5.23319 | 5.23258 | 5.289863 |
| | 0.2 | -0.01156 | 5.40116 | 5.33646 | 5.33653 | 5.33549 | 5.435615 |
| | 0.5 | -0.01029 | 5.72122 | 5.65798 | 5.65812 | 5.65590 | 5.92565 |

## VI. CONCLUSIONS

In this article, we describe the analysis of boundary value problem for third order nonlinear ODEs over an infinite interval arising in axisymmetric flow over a stretching sheet. The semi-numerical and an approximate analytical scheme described here offer some advantages over solutions by HAM, HPM, Adomain decomposition methods and AVIM etc. The convergence of the Dirichlet series methods is given. The results are presented in Tables.

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