Detecting Monotonic Graph over Edge Connectivity Constraint

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***Abstract* - Edge Connectivity parameter of a given graph is widely used in graph mining and finds usage in wide range of applications like transportation problem , Microarray Data , Bioinformatics etc. Based upon the property of a given graph on edge connectivity constraint the decomposition of graph can be avoided, which in turn reduces computational time. The paper focuses at determining whether the given Graph (G) satisfies Monotone property over Edge Connectivity Constraint. It takes into consideration the Maximum degree, Minimum degree and Edge Connectivity values of given Graph (G), based on which the algorithm determines if the given graph satisfies Monotone Property.**

Key Words: Maximum Degree, Minimum Degree, Edge Connectivity.

I. INTRODUCTION

Graph Mining is gaining importance due to the numerous applications that on graph based data. A graph is composed of set of vertices V_G and a set of edges E_G. Each vertex has a label (or a identifier) and each edge
$e_{i,j}$ □ E_G connects the vertices v_i and v_j .In some application the labels are unique and the graph is termed as relational graph whereas if the labels are not unique then the graph is said to be non relational graph. In this we have restricted ourselves to relational graph.

The monotone property of graph plays a major role in skyline approach where in the records returned to the user are the are the ones that are not dominated by any other record, where domination is based on the values of the record. Let p and q be two records, each composed of d attributes. We denote by pi (qi_j) the value of the i-th attribute of p (q) .Record p dominates record q if p is "as good as" q in all attributes and is "better" than q in atleast one attribute. Skyline processing is scale invariant, it does not require a ranking function, it does not require any threshold and can be used as long as long as the data dimensionality is low.

In Skyline approach considering the Edge Connectivity as constraint, Determining of monotone graph will reduce the computational time as the graph need not be decomposed further as the obtained graph will be dominating all its subgraphs over edge connectivity constraints.

The edge connectivity of a given graph signifies the measure of coherence of the given graph. The larger value of edge connectivity signifies that the given graph is more coherent.

II. RELATED WORK AND CONTRIBUTION

There is on-going interest in the research community regarding knowledge discovery from graph data (Cook and Holder 2007).In this section , we briefly present fundamental contribution to our work. There has been significant contribution with regard to skyline approach. The recent contribution is with regard to SkyGraph algorithm (Apostolos N. Papadopoulos et al. 2008) where in the edge connectivity and number of vertices has been taken as constraints. Another Major contribution with regard to constraint based mining is FREQT Algorithm (Jeroen De Knij et al.) Edge Connectivity has been applied as a clustering tool, where clusters are formed by the vertices of a graph G that show a high degree of connectivity (Hartuv and Shamir 2000; Wu and Leahy 1993).Our Work is inspired by the work carried out by (Apostolos N. Papadopoulos et al. 2008).

The problem we study in this work is formally stated as follows: Given a Relational Graph G, we *determine whether the given graph G satisfies Monotonic Constraint over Edge Connectivity Constraint.*

III. PRELIMINARIES

Definition 1 Graphs: A graph G = (V,E) is a pair in which V is a (non-empty) set of vertices or nodes and E is either a set of edges E C {{v,w} | v, w □ v, v □ w} or a set of arcs E C {(v,w) | v, w □ v, v □ w}. In the latter case we call the graph directed.

Definition 2 The edge connectivity, λ *(G)*, of a connected Graph *G* is the minimum number of edges whose removal results in two connected subgraphs.

Definition 3 Maximum Degree (Δ) of a given Graph (G) is defined as the largest degree over all the vertices.

Definition 4 Minimum degree (δ) of a given Graph (G) is defined as the smallest degree over all the vertices.

Definition 5 Monotonic Constraint: A monotonic Constraint is a constraint C_m such that for all Subgraphs H derived from a Graph G satisfies C_m if H satisfies it.

Monotonic Graph over Edge Connectivity Constraint implies that the set of subgraphs (H) obtained by decomposing the graph (G) will always contain the value of the edge connectivity less than that of Graph (G).

IV.MONOTONIC AND ANTI-MONOTONIC GRAPHS

Lemma One

If the graph (G) having edge connectivity one satisfies monotonic property if and only if it is a tree.

Proof: Consider a Graph G, Let the edge connectivity of Graph (G) be 1 i.e λ (G) = 1.

Since the given graph is tree its edge connectivity is always 1 and the edge connectivity of its induced subgraph is either 1 or 0.

Hence a Graph (G) satisfies monotonic property over edge connectivity constraint.

Lemma Two

If Δ is the Maximum Degree and δ is the minimum Degree of a given graph (G) then the edge connectivity of the subgraph (H) obtained from the given graph (G) is always less than or equal to Maximum Degree.

Proof: i) The Subgraph (H) obtained from given Graph (G) can have Minimum Degree at most Δ.

ii) The edge connectivity of (H) cannot be greater than Minimum Degree (δ).

Hence Edge Connectivity of H is at most Δ.

Lemma Three

Consider a Graph G Having maximum degree = Δ, Minimum Degree = δ and Edge Connectivity = λ_G .Let H be set of all induced subgraphs obtained from Given Graph G having Edge Connectivity $\lambda_{H.}$

If maximum degree is equal to Minimum Degree (Δ = δ) then the edge connectivity of all the induced subgraph ($\lambda_{H.}$) is always less than the edge connectivity λ_G of Given Graph. The Given Graph does not contain any Bridges.

Proof: Since the Maximum Degree (Δ) = Minimum Degree (δ).

Therefore Edge Connectivity of given graph λ_G is less than equal to Maximum Degree (Δ) or Minimum Degree (δ).

The Edge Connectivity λ_H of Subgraph (H) is always less than equal to Maximum Degree (Δ) from Lemma One.

Hence the given graph will always satisfy monotone constraint over edge connectivity.

Lemma Four

Consider a Graph G Having maximum degree = Δ, Minimum Degree = δ and Edge Connectivity = λ_G .Let H be set of all induced subgraphs obtained from Given Graph G having Edge Connectivity $\lambda_{H.}$

If the Edge Connectivity λ_G of Graph (G) is less than Minimum Degree (δ) and there exists clique with Maximum degree equal to δ then there exists Subgraph whose edge connectivity is greater then edge connectivity of Graph (G), Hence Graph Does not Satisfy Monotonic Property.

Proof: If λ_G is edge connectivity of Graph G and δ is minimum degree of Graph G and Edge Connectivity λ_G less than Minimum Degree δ of graph G and there exist a clique whose minimum degree is equal to δ, then edge connectivity (λ_H) of clique is equal to δ.

This Implies λ_H is greater than $\lambda_{G.}$

Hence a given Graph (G) satisfies anti-monotonic property over edge connectivity constraint.

Lemma Five

Consider a Graph G Having maximum degree = Δ, Minimum Degree = δ and Edge Connectivity = λ_G .Let

H be set of all induced subgraphs obtained from Given Graph G having Edge Connectivity $\lambda_{H.}$

If the Edge Connectivity λ_G of Graph (G) is equal to Minimum Degree (δ) and minimum degree is less than equal to n where n is less than Maximum Degree (∆) and there exists clique with Maximum degree equal to n+1 then there exists Subgraph whose edge connectivity is greater then edge connectivity of Graph (G), Hence Graph Does not Satisfy Monotonic Property.

Proof: Let λ_G be edge connectivity of Graph G and δ be minimum degree of Graph G.

If λ_G = δ and δ is less than equal to n and ∆ is greater than n and there exist a clique with minimum degree equal to n + 1, This implies Edge Connectivity (λ_H) of clique is greater than δ.

Hence a given Graph (G) satisfies anti-monotonic property over edge connectivity constraint.

V. ALGORITHM

Algorithm MonotoneGraph (G)

Input: G, Initial Input Graph

Output: Monotone Graph (G)

1. Initialize Graph (G)
2. If Graph (G) contains a bridge && G is a Tree
3. Then G is Monotone
4. Else Calculate Maximum Degree (∆) and Minimum Degree (δ).
5. Endif
6. If Maximum Degree = = Minimum Degree
7. Then G is Monotone
8. Else Calculate Edge Connectivity (λ)
9. Endif
10. If Maximum Degree □ Minimum Degree
11. If Minimum Degree = = Edge Connectivity && Minimum Degree = = n, n < Maximum Degree

If There Does Not Exist Clique with degree (n+1)

13. Then G is Monotone
14. Endif
15. Else If Minimum Degree > Edge Connectivity
16. If there Exist no Clique with degree (δ).
17. Then G is Monotonic
18. Endif
19. Endif

V.I EXPLANATION

Line 2 to Line 5: Here the given Graph (G) is checked for bridge. If it contains bridge then it is checked if the given graph is tree. If it satisfies above condition then the given graph satisfies monotonic property. Else calculate maximum degree and Minimum Degree of the Graph (G).

Line 6 to Line 9: Here if the Maximum Degree of the Graph (G) equals Minimum Degree of Graph (G) then the given Graph satisfies the monotonic property. Else calculate the Edge Connectivity of the given graph.

Line 10 to Line 14: Here if the Maximum degree of given graph is not equal to Minimum Degree then the algorithm checks for the following condition:

If minimum degree equal to Edge Connectivity and assigns variable n for the value of minimum degree.

Determine if there exist a clique with degree n+1, If There exist no clique with degree n+1 then the given graph satisfies monotonic property.

Line 15 top Line 18: If Minimum degree of a given graph is greater than Edge Connectivity then check for the following condition:

Determine if there exist a clique with degree equal to minimum degree, If There exist no clique with degree equal to minimum degree then the given graph satisfies monotonic property.

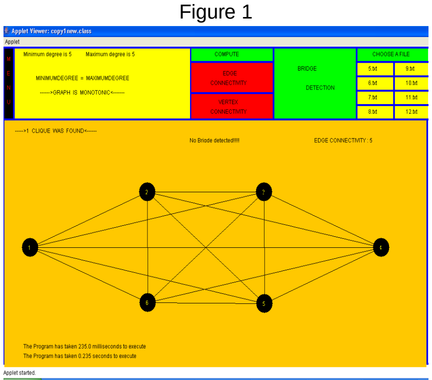
VI. PERFORMANCE EVALUATION

The software project aims at providing Graphical User Interface for various graph mining algorithms. The software has been implemented in JAVA (jdk1.5.0 or jdk1.6.0_12) and the graphs and their corresponding attributes are displayed to the user using the JAVA applet-viewer. All the graph algorithms have been performed on an Intel Core i3 processor at 3.20GHz, with 3GB RAM running Windows XP. The software basically checks for graph parameters such as order, size, etc. that are stored or saved in a foreign source (i.e. in a text file) and using this information about the graph, various graph attributes such as vertex/edge connectivity, clique detection, time complexity, etc. are calculated and displayed on the user interface. The software aims at investigating performance by varying the graph parameters and also keeps a track on the time taken to evaluate a graph (i.e. the time complexity of the software).

The various phases of the software for evaluating and displaying a particular graph scanned are as follows: Scan the text file, which the user chooses on the interface (by clicking the corresponding file link) and store the graph data to the corresponding variables. Using these variables, perform the following task and calculate the following graph attributes:
Define and store a particular path in variables, so as to draw and display the graph, using the information from these variables, on the user interface.
Calculate the number of bridges present in the graph
1. Calculate the edge connectivity.
2. Calculate the cliques detected.
3. Check if the given graph is a tree or not.
4. Calculate the minimum and maximum degree of the graph.
5. Check if the graph is monotonic, using information of edge connectivity, cliques, tree information.
These graph attributes are then displayed to the user.

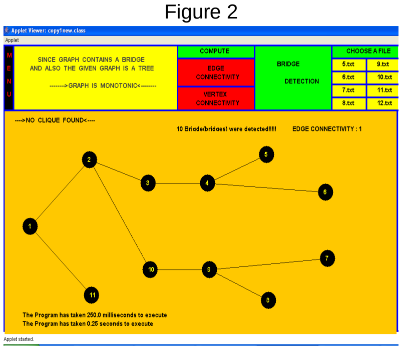
VII. Figures and Results

Figure 1



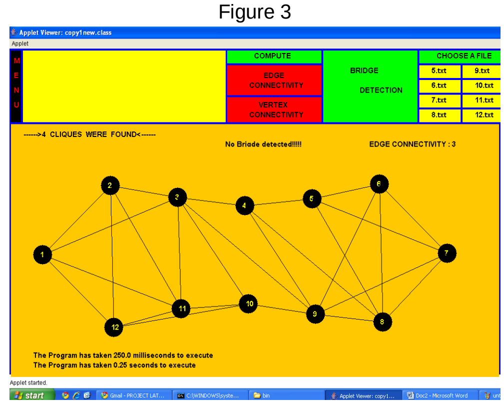
The above window shows the software detecting a clique. The above graph is monotonic since, Minimum degree of the graph = Maximum degree of the graph=5.

Figure 2



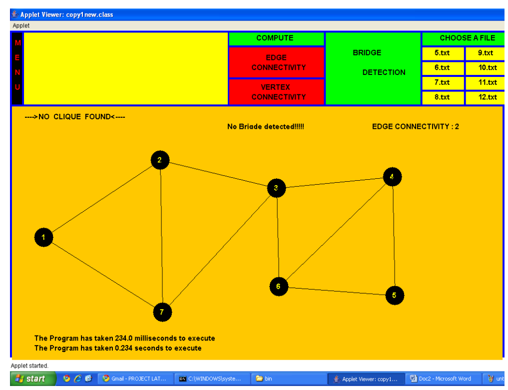
The above window shows the software detecting a Tree. The above graph is monotonic since, its edge connectivity is one and it is a tree.

Figure 3



In the above window, minimum degree (δ) > edge connectivity (λ) and clique degree is at-most = minimum degree i.e. 4 > 3 and 3 <= 4 (Condition Satisfied)

Figure 4



In the above window, minimum degree (δ) = edge connectivity (λ) and Minimum degree == edge connectivity and (min-degree = n && n < max-degree) and Clique-degree is at most = n. i.e. 2 = 2 and n = 2 && n < 4 and 0 <= 2. (Condition Satisfied)

VII CONCLUSION

In this paper we proposed a novel way to detect if the given graph satisfies monotonic property over edge connectivity constraint. We developed an efficient algorithm MonotoneGraph (G) which determines whether the given graph is Monotone over Edge

Connectivity Constraint. Software using Java Programming Language has been designed, which assists in determining the above property of graph .It helps in determining the value of Maximum Degree, Minimum Degree, Edge Connectivity of given graph (G) and Determine if there exist a clique in given graph (G).

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